Tutorial #12

MAT 188 – Linear Algebra I – Fall 2015

SOLUTIONS

Problems (Please note these are from Holt's Linear Algebra Text)

6.6 - # 25 Find the general solution of the system

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \frac{1}{10}\begin{pmatrix} -1 & 2 \\ -3 & -6 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

Solution As we've seen already, we only need the eigenvalues and eigenvectors of the above matrix. Thus we compute the characteristic polynomial

$$P(\lambda) = \det(1\lambda - A) = \frac{1}{100}\begin{vmatrix} 10\lambda+1 & -2 \\ 3 & 10\lambda+6 \end{vmatrix} = \lambda^2 + \frac{7}{10}\lambda + \frac{12}{100} = \left(\lambda + \frac{3}{10}\right)\left(\lambda + \frac{4}{10}\right)$$

We easily see the eigenvalues are given by $\lambda = -4/10, -3/10$, we compute the eigenvectors via $(A - 1\lambda)u = 0$:

$$\lambda_1 = -\frac{3}{10} \implies \ker\begin{pmatrix} -2 & -2 \\ 3 & 3 \end{pmatrix} = \operatorname{span}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \implies \vec{\lambda}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\lambda_2 = -\frac{4}{10} \implies \ker\begin{pmatrix} -3 & -2 \\ 3 & 2 \end{pmatrix} = \operatorname{span}\begin{pmatrix} 2 \\ -3 \end{pmatrix} \implies \vec{\lambda}_2 = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

Thus the solution to the system is given by

$$\boxed{y = c_1\vec{\lambda}_1 e^{\lambda_1 t} + c_2\vec{\lambda}_2 e^{\lambda_2 t} = c_1\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{-3t/10} + c_2\begin{pmatrix} 2 \\ -3 \end{pmatrix}e^{-4t/10} \quad c_1, c_2 \in \mathbb{R}}$$

8.1 - #32 Find a basis for $S^\perp$ for

$$S = \operatorname{span}\left\{\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}\right\}$$

Solution We want a vector u such that $e_1 \cdot u = e_2 \cdot u = 0$. So we simply take the cross product of e_1 and e_2,

$$u = e_1 \times e_2 = \det\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ 2 & -3 & 2 \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \\ -1 \end{pmatrix} \implies S^\perp = \operatorname{span}\left\{\begin{pmatrix} 7 \\ 4 \\ -1 \end{pmatrix}\right\}$$

Quiz Find the general solution of the system

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$